(۱)

$$\int \frac{\cos x}{\sin x + 1}\,dx \qquad \begin{cases} u = \sin x + 1 \\ du = \cos x\,dx \end{cases}$$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\sin x + 1| + C$$

(۲)

$$\int \frac{1 + \ln x}{x}\,dx \qquad \begin{cases} u = 1 + \ln x \\ du = \frac{dx}{x} \end{cases}$$

$$= \int u\,du = \frac{u^2}{2} + C = \frac{(1+\ln x)^2}{2} + C$$

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اسفند

(۳)

$$\int x^2 (2x-2)^2\,dx = \int x^2 (4x^2 - 8x + 4)\,dx$$

$$= \int (4x^4 - 8x^3 + 4x^2)\,dx = \frac{4}{5}x^5 - \frac{8}{4}x^4 + \frac{4}{3}x^3 + C$$

(۴)

$$\int \frac{1}{x^3}(\sqrt{x} + 2x)\,dx = \int \left(\frac{\sqrt{x}}{x^3} + \frac{2x}{x^3}\right) dx$$

$$= \int \left(x^{-\frac{5}{2}} + \frac{2}{x^2}\right) dx = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} - \frac{2}{x} + C$$

$$= \frac{-2}{3x\sqrt{x}} - \frac{2}{x} + C$$

(۵)

$$\int \frac{\sqrt{x}+x}{\sqrt[3]{x^2}}\,dx = \int \left(\frac{\sqrt{x}}{\sqrt[3]{x^2}} + \frac{x}{\sqrt[3]{x^2}}\right)dx$$

$$= \int \left(\frac{x^{\frac{1}{2}}}{x^{\frac{2}{3}}} + \frac{x}{x^{\frac{2}{3}}}\right)dx = \int \left(x^{\frac{1}{2}-\frac{2}{3}} + x^{1-\frac{2}{3}}\right)dx$$

$$= \int \left(x^{\frac{-1}{6}} + x^{\frac{1}{3}}\right)dx = \frac{x^{-\frac{1}{6}+1}}{-\frac{1}{6}+1} + \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$$

$$= \frac{6}{5}x^{\frac{5}{6}} + \frac{3}{4}x^{\frac{4}{3}} + C$$

(۶)

$$\int (x^2-1)^3 x\,dx = \int (x^9 - 3x^6 + 3x^3 - 1)\,dx$$

$$= \frac{x^{10}}{10} - \frac{3}{7}x^7 + \frac{3}{4}x^4 - x + C$$

(۷)

$$\int (x^2+\ln x)^5 \left(2x+\frac{1}{x}\right)dx$$

$$\begin{cases} u = x^2+\ln x \\ du = \left(2x+\frac{1}{x}\right)dx \end{cases}$$

$$= \int u^5\,du = \frac{u^6}{6}+C$$

$$= \frac{(x^2+\ln x)^6}{6}+C$$

(۸)

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$\begin{cases} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx \end{cases}$$

$$= \int \sin u \;(2du) = -2\cos u + C$$

$$\int x^5 \sqrt[3]{(1+x^2)^2}\,dx = \int x^3 x^2 \sqrt[3]{(1+x^2)^2}\,dx$$

$$\begin{cases} u = x^3 \\ du = 3x^2 dx \end{cases}$$

$$= \int u \sqrt[3]{(1+u)^2}\left(\frac{du}{3}\right)$$

$$\begin{cases} v = 1+u \\ dv = du \end{cases}$$

$$= \int (v-1)\sqrt[3]{v^2}\left(\frac{dv}{3}\right)$$

$$= \frac{1}{3}\int (v-1)\,v^{\frac{2}{3}}\,dv$$

$$= \frac{1}{3}\int \left(v^{\frac{5}{3}} - v^{\frac{2}{3}}\right)dv$$

$$= \frac{1}{3}\,\frac{v^{\frac{5}{3}+1}}{\frac{5}{3}+1} - \frac{v^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C = \frac{1}{8}v^{\frac{8}{3}} - \frac{3}{5}v^{\frac{5}{3}} + C$$

$$= \frac{1}{8}(1+u)^{\frac{8}{3}} - \frac{3}{5}(1+u)^{\frac{5}{3}} + C$$

$$= \frac{1}{8}(1+x^3)^{\frac{8}{3}} - \frac{3}{5}(1+x^3)^{\frac{5}{3}} + C$$

(۹)

$$\int \sqrt[3]{(\sin x - \ln x)}\ (\cos x - \frac{1}{x})\,dx$$

$$\begin{cases} u = \sin x - \ln x \\ du = (\cos x - \frac{1}{x})\,dx \end{cases}$$

$$= \int \sqrt[3]{u}\,du = \int u^{\frac{1}{3}}\,du$$

$$= \frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}u^{\frac{4}{3}} + C$$

$$= \frac{3}{4}(\sin x - \ln x)^{4/3} + C$$

(۱۰)

$$\int \cos x\, e^{\sin x}\,dx$$

$$\begin{cases} u = \sin x \\ du = \cos x\,dx \end{cases}$$

$$= \int e^{u}\,du = e^{u} + C = e^{\sin x} + C$$

(۱۱)

$$\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \frac{u^3}{3} + C$$

$$\begin{cases} u = \ln x \\ du = \frac{dx}{x} \end{cases}$$

$$= \frac{(\ln x)^3}{3} + C$$

(۱۲)

$$\int (2x^2-1)^9 x\,dx$$

$$\begin{cases} u = 2x^2-1 \\ du = 4x\,dx \end{cases}$$

$$= \int u^9 \left(\frac{du}{4}\right) = \frac{1}{4}\int u^9\,du$$

$$= \frac{1}{4}\,\frac{u^{10}}{10} + C = \frac{u^{10}}{40} + C = \frac{(2x^2-1)^{10}}{40} + C$$

(۱۳)

$$\int e^{4x+1}\,dx = \int e^{4x} e\,dx = e\int e^{4x}\,dx$$

$$= \frac{e}{4}\,e^{4x} + C = \frac{e^{4x+1}}{4} + C$$

(۱۴)

$$\int e^x \sqrt{(1+\cos(e^x))^5}\ \sin(e^x)\,dx \qquad \begin{cases} u = 1+\cos e^x \\ du = -e^x \sin e^x dx \end{cases}$$

$$= \int \sqrt{u^5}\ (-du) = -\int u^{\frac{1}{5}} du$$

$$= -\frac{u^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C = -\frac{u^{\frac{6}{5}}}{\frac{6}{5}} + C = -\frac{5}{6} u^{\frac{6}{5}} + C$$

$$= -\frac{5}{6}\left(1+\cos(e^x)\right)^{\frac{6}{5}} + C$$

(۱۵)

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx \qquad \begin{cases} u = 1+\sqrt{x} \\ du = \frac{dx}{2\sqrt{x}} \end{cases}$$

$$= \int \sqrt{u}\ (2du) = 2\int u^{\frac{1}{2}} du$$

$$= 2\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = 2\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{4}{3} u^{\frac{3}{2}} + C$$

$$= \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2} + C$$

(۱۶)

$$\int \frac{x^2}{\sqrt{x^5+1}}\,dx \qquad \begin{cases} u = x^5+1 \\ du = 5x^2\,dx \end{cases}$$

$$= \int \frac{\left(\frac{du}{3}\right)}{\sqrt{u}} = \frac{1}{3}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{3}\,\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{1}{3}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}\sqrt{u} + C$$

$$= \frac{2}{3}\sqrt{x^5+1} + C$$

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(۱۷)

$$\int \frac{x\cos x^2}{\sin x^2}\,dx \qquad \begin{cases} u = \sin x^2 \\ du = 2x\cos x^2\,dx \end{cases}$$

$$= \int \frac{\left(\frac{du}{2}\right)}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|\sin x^2| + C$$

(۱۸)

$$\int x\left(x+\cos(x^2)\right)dx$$

$$=\int\left[x^2+x\cos(x^2)\right]dx=\int x^2dx+\int x\cos(x^2)\,dx$$

$$=\frac{x^3}{3}+\int x\cos(x^2)\,dx$$

$$\begin{cases} u=x^2 \\ du=2x\,dx \end{cases}$$

$$=\frac{x^3}{3}+\int \cos u\left(\frac{du}{2}\right)$$

$$=\frac{x^3}{3}+\frac{1}{2}\sin u+C=\frac{x^6}{6}+\frac{1}{2}\sin(x^2)+C$$

(۱۹)

$$\int \frac{\sin(2x)}{1+\cos^2 x}\,dx$$

$$\begin{cases} u=1+\cos^2 x \\ du=-2\cos x\sin x\,dx \\ du=-\sin 2x\,dx \end{cases}$$

$$=\int \frac{-du}{u}=-\ln|u|$$

$$=-\ln|1+\cos^2 x|+C$$

(۲۰)

$$\int \frac{dx}{(\text{Arc}\sin x)^3\sqrt{1-x^2}}$$

$$\begin{cases} u = \text{Arc}\sin x \\ du = \dfrac{dx}{\sqrt{1-x^2}} \end{cases}$$

$$= \int \frac{du}{u^3} = \int u^{-3}\,du$$

$$= \frac{u^{-2}}{-2} + C = -\frac{1}{2u^2} + C = -\frac{1}{2(\text{Arc}\sin x)^2} + C$$

(۲۱)

$$\int \text{Arc}\tan x\,dx$$

$$\begin{cases} u = \text{Arc}\tan x \\ dv = dx \end{cases} \rightarrow \begin{cases} du = \dfrac{dx}{1+x^2} \\ v = x \end{cases}$$

$$= x\,\text{Arc}\tan x - \int \frac{x}{1+x^2}\,dx$$

$$\begin{cases} u = 1+x^2 \\ du = 2x\,dx \end{cases}$$

$$= x\,\text{Arc}\tan x - \int \frac{\left(\frac{du}{2}\right)}{u}$$

$$= x\,\text{Arc}\tan x - \frac{1}{2}\,\text{Ln}|u| + C = x\,\text{Arc}\tan x - \frac{1}{2}\,\text{Ln}|1+x^2| + C$$

(۲۲)

$$\int x \operatorname{Arccos} x\,dx$$

$$\begin{cases} u = \operatorname{Arccos} x \\ dv = x\,dx \end{cases} \quad \begin{cases} du = \dfrac{-dx}{\sqrt{1-x^2}} \\ v = \dfrac{x^2}{2} \end{cases}$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \int \frac{x^2 dx}{2\sqrt{1-x^2}}$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{2}\int \frac{x^2 dx}{\sqrt{1-x^2}}$$

$$\begin{cases} x = \sin t \\ dx = \cos t\,dt \end{cases}$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{2}\int \frac{\sin^2 t \cos t\,dt}{\sqrt{1-\sin^2 t}}$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{2}\int \sin^2 t\,dt$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{4}\int \frac{1-\cos 2t}{2}\,dt$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{8}\left(t - \frac{1}{2}\sin 2t\right) + C$$

$$= \frac{x^2}{2}\operatorname{Arccos} x + \frac{1}{8}\left(\operatorname{Arcsin} x - \frac{1}{2}\sin\left(2\operatorname{Arcsin} x\right)\right) + C$$

(۲۳)

$$\int x^2 \cos x\, dx$$

x^2	$+$	$\cos x$
$2x$	$-$	$\sin x$
2	$+$	$-\cos x$
0		$-\sin x$

$$= x^2\sin x + 2x\cos x - 2\sin x + C$$

(۲۴)

$$\int x e^{3x} dx$$

x	$+$	e^{3x}
1	$-$	$\frac{1}{3}e^{3x}$
0		$\frac{1}{9}e^{3x}$

$$= \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$$

(۲۵)

$$\int x\tan x\, dx =$$ تابع اولیه ندارد !!!

(۲۶)

$$\int \sin(\text{Arctan}x)\, dx$$

$$\begin{cases} t = \text{Arctan}x \\ x = \sin t \\ dx = \cos t\, dt \end{cases}$$

$$= \int \sin t \cos t\, dt$$

$$= \frac{1}{2}\int 2\sin t\cos t\, dt = \frac{1}{2}\int \sin 2t\, dt = \frac{-1}{4}\cos 2t + C$$

$$= -\frac{1}{4}\cos\left(2(\text{Arctan}x)\right) + C$$

(۲۷)

$$\int \ln x\,dx \qquad \begin{cases} u = \ln x \\ dv = dx \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = x \end{cases}$$

$$= x\ln x - \int dx = x\ln x - x + C$$

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(۲۸)

$$\int x \ln\left(1+\frac{1}{x}\right) dx \qquad \begin{cases} u = \ln\left(1+\frac{1}{x}\right) \\ dv = x\,dx \end{cases}$$

$$\begin{cases} du = \dfrac{-\frac{1}{x^2}}{1+\frac{1}{x}}\,dx \\ v = \frac{x^2}{2} \end{cases}$$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) - \int \frac{-\frac{1}{2}}{1+\frac{1}{x}}\,dx$$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\int \frac{x}{x+1}\,dx$$

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$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\int \frac{x+1-1}{x+1}\,dx$$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\left[x - \ln|x+1|\right] + C$$

(۳۰)

$$\int \text{Arcsinh}\, x\, dx$$

$$\begin{cases} u = \text{Arcsinh}\, x \\ dv = dx \end{cases} \rightarrow \begin{cases} du = \dfrac{dx}{\sqrt{1+x^2}} \\ v = x \end{cases}$$

$$= x\,\text{Arcsinh}\, x - \int \frac{x\,dx}{\sqrt{1+x^2}}$$

$$\begin{cases} w = 1+x^2 \\ dw = 2x\,dx \end{cases}$$

$$= x\,\text{Arcsinh}\, x - \int \frac{\left(\frac{dw}{2}\right)}{\sqrt{w}}$$

$$= x\,\text{Arcsinh}\, x - \frac{1}{2}\int w^{-\frac{1}{2}}\,dw$$

$$= x\,\text{Arcsinh}\, x - \frac{1}{2}\left[2\sqrt{w}\right] + C$$

$$= x\,\text{Arcsinh}\, x - \sqrt{1+x^2} + C$$